THE DESIGN OF PROBABILITY THEORY

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Abstract

The problem of coping with uncertainty in the real world is central to all science. As a result of the work by many people a fairly satisfactory solution has been achievedalthough much controversy still remains. The solution involves, first, a scheme that allows one to represent any provisional state of knowledge, and second, a procedure that allows us to revise our beliefs as we acquire new information.

This tutorial is mostly concerned with the first stage—the design of probability theory—as pioneered by Cox. About the second stage—the updating problem—our discussion will be brief, we will only make some brief remarks on Bayes rule and its limitations.

I will follow a very pragmatic approach. Probability theory is neither true nor false; it is designed to be useful, to achieve a certain purpose, to work. Just as in engineering, one is satisfied that a solution works when it performs according to some desired "design specifications" or "design criteria". In engineering there may be many solutions that work fine—they perform the desired function, and in the end, that is all we care about. What is remarkable about the Cox approach is that the design specifications—consistency, universality—are totally restrictive. There is a unique way to handle degrees of belief and this is probability theory.

I will address some of the criticisms that have been raised against the Cox approach. Are these degrees of belief, or plausibility, or credibility, or even degrees of implication? Is there a difference? Should we use a single real number to measure a degree of belief? Can beliefs be compared? Are the Cox design criteria obvious? Are there counter examples to Cox?

Rather than justifying Cox's choices I demonstrate their robustness. I make a different choice of design criteria and derive probability theory as the unique (up to regraduations) consistent representation of the Boolean AND and OR operations.

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