ON THE NOTION OF FAIR GAMES AND BERNOULLI'S CONCEPT OF MORAL EXPECTATION

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Abstract

It is commonly believed that Daniel Bernoulli's resolution of the St Petersburg problem provides an estimate of the 'fair' entrance fee for that game. This fundamental misunderstanding of Bernoulli's treatment of the problem didn't escape Edwin Jaynes' attention: *In one of the best known books on probability theory (Feller, 1950, p. 199), Daniel Bernoulli's resolution of the famous St Petersburg paradox is rejected without even being described, except to assure the reader that he 'tried in vain to solve it by the concept of moral expectation'. ... Reading Feller, one finds that he 'resolved' the paradox merely by defining and analyzing a different game.*

For asymmetrical games with two players, it can be quite difficult to come up with a criterion of game fairness everyone finds satisfactory. However, in the case of perfectly symmetrical games, there can be no doubt that such games are fair. Nevertheless, as we demonstrate in this paper, Bernoulli's approach to estimating the 'fair' entrance fee can yield different results for different players even in cases where asymmetry of the game is purely superficial. That shows clearly that the essence of Bernoulli's resolution of the St Petersburg paradox is not about calculating the 'fair' entrance fee at all.

One of the principal objections to Bernoulli's resolution of the St Petersburg paradox is that of justification for the logarithmic assignment of utility: the choice of the logarithm doesn't seem to follow from first principles of probability theory. Indeed, why the utility of amount of money, M, or the 'moral value' as Bernoulli called it, should be taken proportional to $log(M)$? Why the utility is not to be taken proportional to $(M-1)^{1/3}$, or $arctan(M-1)$, which, after all, has an additional advantage of being bounded? The choice of the logarithmic assignment of utility appears as an ad-hoc device with all earmarks of such devices.

In this paper we show that Bernoulli's result can be obtained without resorting to the concept of moral expectation. Using the classical mathematical expectation in combination with arguments dictated by common sense, we obtain a functional equation for the entrance fee in the St Petersburg game. The solution of the equation yields entrance fee as a function of the total fortune of the player, which is in close agreement with that obtained by Bernoulli based on his concept of moral expectation.